AIP
All Island Project

Single Electricity Market

Loss of Load Probability Curve for Capacity Payment Mechanism

Consultation Paper

February 2007

AIP/SEM/07/10

TABLE OF CONTENTS

1. INTRODUCTION

On 22 December 2006 the Commission for Energy Regulation and the Northern Ireland Authority for Energy Regulation ("the Regulatory Authorities") published a decisions paper entitled *"Capacity Payment Factors"*[1]. This paper set out the Regulatory Authorities' response to the comments received and presented the conclusions of the Regulatory Authorities' in the matters addressed in the preceding consultation paper[2].

One of the issues addressed in the decisions paper was the determination of Loss of Load Probabilities (LOLP) values. These values are used to distribute the monthly pots of Variable and Ex-Post Capacity Payments over the trading periods in each month. At the time the consultation document was issued, work was ongoing by the TSOs for the Regulatory Authorities to model the impact of different LOLP curves, and therefore no particular curves were recommended.

This document discusses the determination of LOLP curves and identifies the approach preferred by the Regulatory Authorities. The structure of this document is as follows:

> Section 2 sets out the background to the development of the Capacity Payment Mechanism (CPM);
>
> Section 3 considers different methodologies to derive LOLP curves;
>
> Section 4 discusses the data requirements;
>
> Section 5 discusses the necessity of separate LOLP curves for Variable and Ex-Post Capacity Payments;
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> Section 6 summarises the proposals and calls for views on specific issues.

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1 http://www.allislandproject.org/2006/AIP-SEM-231-06.doc
2 http://www.allislandproject.org/2006/AIP-SEM-161-06.pdf

2. BACKGROUND

On 15th July 2005 the Regulatory Authorities issued a paper titled "*Capacity Payment Mechanism and Reserve Charging High Level Decision paper*"[3] in which the Regulatory Authorities stipulated their intention to develop a fixed revenue capacity payment mechanism which would provide a degree of financial certainty to generators under the new market arrangements and a stable year-to-year pattern of capacity payments.

The principles outlined in the July 2005 paper were incorporated into the design of the CPM in the all-island Trading and Settlement Code (T&SC) and on 21st December 2005, the Regulatory Authorities published a draft version (version 0.10) of the proposed T&SC for the SEM, with comments invited by 20th January 2006. Subsequent to the publication of this document the Regulatory Authorities determined that a more detailed consideration of the comments received on the design of the CPM was required and on 3rd March 2006 the Regulatory Authorities issued a further consultation paper[4]. Following a further open forum discussion the Regulatory Authorities issued a Decision document in July 2006[5] which described the selected CPM and attached a set of associated changes required to the T&SC version 1.0.

On 5 October 2006 the Regulatory Authorities issued a further consultation[6] on a number of further detailed matters relating to the design of the CPM which had not been addressed by the consultation issued in March 2006. Most recently, on 22 December 2006 the Regulatory Authorities published a decisions paper entitled *"Capacity Payment Factors"*[7]. One of the remaining issues was the determination of Loss of Load Probability (LOLP) values, which will be addressed in this paper.

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3 http://www.allislandproject.org/AIP-SEM-53-05.pdf
4 http://www.allislandproject.org/2006/AIP-SEM-15-06.pdf
5 http://www.allislandproject.org/2006/AIP-SEM-95-06.pdf
6 http://www.allislandproject.org/2006/AIP-SEM-161-06.pdf
7 http://www.allislandproject.org/2006/AIP-SEM-231-06.doc

3. METHODOLOGY

3.1. Introduction

A loss of load is defined as a system failure to match demand with available capacity. The Loss of Load Probability (LOLP) is the probability that a loss of load occurs for a given combination of system demand and generators' availabilities. As a formula:

$$LOLP = \Pr(S < D)$$

where *Pr()* is a probability function, *S* is aggregate supply, and *D* is system demand. In turn, the Forced Outage Probability (FOP) is the probability that a generator is out of service for reasons other than scheduled maintenance. This means that the available capacity of the system is the aggregate of generators' availabilities, each of them dependent on its FOP. The available capacity of generator *i* (s_i) can then be presented as:

$$s_i = \begin{cases} 0, & \Pr(s_i = 0) = FOP_i \\ c_i & \Pr(s_i = c_i) = 1 - FOP_i \end{cases}$$

where c_i is the capacity and FOP_i is the forced outage probability of plant *i*. The aggregate available capacity (*S*) of all *n* generators is then:

$$S = \sum_{i=1}^{n} s_i$$

The Regulatory Authorities proposed to use a Margin *vs.* LOLP look-up table. This assumes that the probability of a loss of load at a certain margin is constant regardless of the level of system demand or plant mix. The margin *M* can be expressed as *(C-D)*, where *C* is the aggregate capacity. The LOLP can then be presented as:

$$LOLP = \Pr(S < D) = \Pr(S < C - M) = \Pr(C - S > M)$$

In other words, the probability that demand exceeds available capacity is equal to the probability that aggregate outage exceeds the margin. This simple transformation means that a derived probability distribution function of available capacity is inverted to become the probability distribution function of aggregate outage. In the following section, several methodologies are presented to derive a probability distribution function of aggregate available supply *S*.

3.2. Methodologies

3.1.1. Direct calculation

The direct calculation methodology would derive the LOLP for all possible combinations of forced outages. For example, the formulae below calculate the probability that (a) all generators are out of service, and (b) all generators are available:

$$(a) \quad \Pr(S=0) = FOP_1 \times FOP_2 \times \ldots \times FOP_n = \prod_{i=1}^{n} FOP_i$$

$$(b) \quad \Pr\left(S = \sum_{i=1}^{n} C_i\right) = \prod_{i=1}^{n}(1 - FOP_i)$$

where Π is the product operator. For every combination of outage, one can calculate its probability of occurring – as a function of FOPs – and the aggregate available capacity.

The disadvantage of this methodology is the large number of calculations. As there are two possible working states – operational or out of service – the number of combinations equals 2^n. For example, there are 2^{60} ($1.15*10^{18}$) combinations for 60 generating plants.

3.1.2. Approximation

The FOP (and thus *s*) of each generator is a binomial random variable. A normal probability distribution can provide a good approximation of *S* if both the number of generators and the individual probabilities are sufficiently large. The latter is not the case, as a typical FOP value is 0.05 (5%). This will cause problems, particularly when estimating probabilities at the tail-end of the distribution[8]. An added complication is the difference in plant size.

[8] A useful rule of thumb is that the interval μ±3σ should lie within the range 0 to *n*, where μ is the average number of outages, σ is the standard deviation and *n* is the number of generators. At a typical FOP of 5%, the normal approximation can be used when there are at least 171 generators. When the FOP is 4%, this number increases to 216.

3.1.3. Simulation

Another approach is to add to the available capacity of a generating plant a random element that is in line with its FOP. These random available capacities of all generators are aggregated into the total available capacity. A distribution function of total available capacity can then be derived from a large number of random runs. This method could be a lot faster than the direct calculation, as it does not require specifying each possible combination of outage. However, the number of runs should be much larger than the number of possible combinations to ensure that the probabilities at the tail-end of the distribution are estimated correctly. For example, the least likely scenario is that all generators are out of service. The probability of this scenario is 0.05^{60} ($8.67*10^{-79}$) when there are 60 generators and a typical FOP of 5%. In other words, this scenario is expected to occur only once in $1.15*10^{78}$ random runs. Even then, there is a 60% chance that this scenario does not occur or occurs more than once.

3.1.4. *Stacking* methodology

Villagarcía[9] developed a fast algorithm to compute the exact distribution of the aggregate available supply *S*. A subset Z_k is defined such that:

$$Z_k = \sum_{i=1}^{k} s_i$$

where *k* is the number of generators in the subset, $Z_1 = s_1$ and $Z_n = S$. The probability distribution function $G_k(x)$ is defined as:

$$G_k(x) = \Pr(Z_k < x)$$

For the first generator, the probability is equal to FOP_1 for all *x* smaller or equal to the capacity of the first generator. Say, there is one generator (100MW capacity and 5% FOP). Whether demand is 60MW or 70MW (i.e. margin is 40MW or 30MW respectively) the loss of load probability is identical and equal to the FOP.

9 Villagarcía, T. (1998). The Use of Consulting Work to Teach Statistics to Engineering Students. *Journal of Statistics Education* 6 (2). http://www.amstat.org/publications/jse/v6n2/villagarcia.html

For the second and sequential generators, the distribution function is *stacked* using the following property of the distribution function:

$$\begin{aligned} G_{k+1}(x) = \Pr(Z_{k+1} < x) &= \Pr(Z_k < x \mid s_{k+1} = 0) \cdot \Pr(s_{k+1} = 0) \\ &\quad + \Pr(Z_k < x - c_{k+1} \mid s_{k+1} = c_{k+1}) \cdot \Pr(s_{k+1} = c_{k+1}) \\ &= G_k(x) \cdot FOP_{k+1} + G_k(x - c_{k+1}) \cdot (1 - FOP_{k+1}) \end{aligned}$$

Say, a second generator (200MW, 4% FOP) is added to the previous example. The aggregate supply distribution function of both generators is as follows[10]:

For x<100 $$\begin{aligned} G_2(x) &= G_1(x) \cdot FOP_2 + G_1(x - c_2) \cdot (1 - FOP_2) \\ & 0.05 \cdot 0.04 + 0.00 \cdot 0.96 = 0.002 \end{aligned}$$

For 100≤x<200 $$\begin{aligned} G_2(x) &= G_1(x) \cdot FOP_2 + G_1(x - c_2) \cdot (1 - FOP_2) \\ & 1.00 \cdot 0.04 + 0.00 \cdot 0.96 = 0.040 \end{aligned}$$

For x≥200 $$\begin{aligned} G_2(x) &= G_1(x) \cdot FOP_2 + G_1(x - c_2) \cdot (1 - FOP_2) \\ & 1.00 \cdot 0.04 + 0.05 \cdot 0.96 = 0.088 \end{aligned}$$

The addition of a third and fourth generator are derived in Appendix A. This process will continue until all generating units are included (i.e. $Z_n = S$). Villagarcía provides syntax in MATLAB to derive the probability distribution function, but it can also be computed in Microsoft Excel. This approach is very fast and efficient. Note that although the LOLP curve from this approach is never increasing, it will not be *continuously* decreasing as there will be "flat" sections whenever a new unit is added.

3.1.5. Alternative Curves

It is possible to define alternative curves to convert margins into parameters for the distribution of the monthly pots of Variable and Ex-Post Capacity Payments. The advantage of such curves is that they can be moulded to provide a desired incentive for availability at times when margins are tight.

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10 $G_1(x)$ is equal to the probability that available capacity s_1 is smaller than x. For x larger than c_1, this probability equals one. For x smaller or equal to zero, this probability equals zero. Similarly, $G_1(x-c_2)$ equals zero for x smaller than c_2.

An example of such a curve is proposed by Garver[11]:

$$R' = \exp\left(-\frac{(P-L)}{m}\right)$$

where *R'* is the risk approximation, *P* is the peak load, *L* is the actual load and *m* is a constant parameter. When the load is at its peak (*P=L*), the risk approximation equals one, i.e. maximum risk. Another advantage of this curve is that it is continuously decreasing with increasing margins for every margin, i.e. *LOLP(x) > LOLP(x+1)* for all *x*, unlike the LOLP curve derived using the stacking method where *LOLP(x) ≥ LOLP(x+1)* for all *x*. These curves cannot be called LOLP curves, as they do not use information on individual generator's FOPs and capacities. Also, it will be difficult to justify the choice of a particular curve or a possible future revision of this curve.

3.3. Flattening the LOLP Curve

Previous modelling has revealed the sensitivity of the allocation of amounts of the CPM pot where margins are relatively high. In the recent decisions paper, the Regulatory Authorities proposed that it might be appropriate to flatten the LOLP curve beyond a certain margin to avoid "spurious" allocations[12]. The effect of this would be a minimum Variable and Ex-Post pot size for trading periods in a particular month where the margin is above a certain level. As a result, there will be a smaller pot left to be distributed over the other (peak) periods.

The flattening of the LOLP curve implies that the probability of a loss of load remains constant beyond a certain margin. It is desirable to have a rationale behind the choice of a cut-off point for flattening the LOLP curve. This would provide a basis for future review of the LOLP curve in light of changes in the system load, demand profile and generation mix.

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11 Garver, L.L. (1966). Effective Load-Carrying Capability of Generating Units. *IEEE Transactions and Apparatus and Systems*. Vol. PAS-85.

12 In this context "spurious" relates to months when there is a relatively large margin for the entire period. In such months it is possible that a relatively large allocation could be made into a single or few Trading Periods where the margin is slightly less than in other periods, even though the difference in margin is quite small.

Appendix B shows the result of flattening the LOLP curve at 2,000MW and 2,500MW respectively[13] against an *unflattened* curved in respect of the distribution of the Variable Capacity Payment pot. The distribution graph is split to demonstrate the impact when margins are large and tight respectively. It shows that given the shape of the LOLP curve, and thus the distribution of capacity payments, the cut-off margin should be quite low to significantly reduce capacity payments at peak times. At the same time, setting the threshold too low could result in payments at near-peak time to become similar to those off-peak or at night time.

The flattening of the LOLP curve will distort the CPM by using an allocation over periods that is disproportionate to the (relative) probability of a loss of load. It should be noted that the effect of a flat LOLP curve on the distribution of capacity payments can also be achieved by increasing the proportion allocated in the fixed capacity pot. Given previous discussions about the relative allocations into the Fixed, Variable and Ex-Post elements the Regulatory Authorities are not minded to pursue such flattening.

[13] The cut-off points of 2,000MW and 2,500MW correspond roughly to margins occurring in 10% and 25% of trading periods respectively.

4. DATA REQUIREMENTS

To derive a LOLP curve for a given set of generators, it is necessary to have capacities and forced outage rates (FOPs) of every generator. The first step, however, is to define the set of generators. Modelling results presented in this paper are driven by the technical input data of the Loop 2 modelling exercise[14].

4.1. Generator Set

The generator set should initially be limited to those generating units eligible to receive capacity payments, since this is the basis of the margin determination. Next the set should consist of the subset of all these units that have a controllable available capacity in the short run. Therefore, it will include the interconnector(s) and energy limited plants, but will exclude wind generation.

4.2. Capacity

Unit Capacities will be requested from generators for the Capacity Requirement and verified by the TSOs. The capacity of an individual unit is not expected to change very frequently. Although some units will receive capacity payments on dispatched quantity (interconnectors, energy limited plants), their full capacity will be entered in the calculation of the LOLP curve as they can be used at any given time when the margin is tight.

4.3. Forced Outage Probabilities

The FOPs can be derived from historic data of individual generator units. It looks at the number of time periods (or incidences and their length) that there was a forced outage in a unit over, say, the previous 5 years. This can then be converted into a probability by dividing it by the number of time periods without a planned outage. Historical data would not be available for new generating units. This means that assumptions will have to be made on their FOPs, possible from data of similar units. Further adjustments can be made when historic data is a-typical, though they need to be considered on a case-by-case basis.

[14] http://www.allislandproject.org/AIP-SEM-124-05.xls

4.4. Changes in Generator Set

The entry or exit of generating units from the market will affect the reliability of the system and as such the probability of a loss of load. It is envisaged that such events will lead to a recalculation of the LOLP curve to be used as from its occurrence. It can be argued that temporary entry or exit, e.g. for unit maintenance purposes, should be treated in the same way.

The generator outage schedule for the next two months should be submitted by the System Operator to the Market Operator according to the data submission requirement in the T&SC (Appendix J.6, Table 45a). This schedule is put together to meet maintenance of generating units whilst maximizing aggregate available capacity given seasonal (peak) load.

Separate LOLP curves can be calculated for different combinations of planned availability. When a unit is removed from the generator set, the LOLP will increase for every level of the margin. However, it depends on the mix of remaining generators in terms of their size and FOP whether the LOLP curve becomes steeper or flatter.

Appendix C presents the results of modelling the distribution of the Variable Capacity Payments using a constant LOLP curve and variable curves derived from a planned outage schedule[15]. There is no difference in January and December; due to the high (peak) demand in these months, no outage is planned. In other months, there are potentially large differences in the hourly pot at peak demand. Overall, using the constant LOLP curve will generally reduce the hourly variable capacity pots at peak times, say the 5% lowest margins, whilst slight increases occur at all other times.

In order to calculate the probability of loss of load at a certain margin, it would be theoretically sound to use a LOLP curve that represents the planned availability of generation. However, there are a number of drawbacks. Firstly, it would increase the number of LOLP curves to be computed, which current systems may not be able to cope with. It would also reduce the transparency of distributing the monthly pots of Variable and Ex-Post Capacity Payments. Finally, the scheduling of planned outage could become controversial as it can indirectly influence the shape of the LOLP curve and thus capacity payments.

[15] The planned outage schedule is taken from the Loop 2 modelling exercise and is representative – but not necessarily accurate – for 2007.

4.5. Data Review and Publication

The System Operators will be responsible for the estimation of the LOLP curve(s), as they are required to send Loss of Load Probabilities for each trading period to the Market Operator under the T&SC (Appendix E.7, Table 19a). It is suggested that the LOLP curve is published at the start of the year or after a periodic review.

The LOLP curve can be periodically reviewed using the latest data on generator mix, capacities and forced outage probabilities. This is particularly the case for an addition of a new generating unit or removal of an existing generating unit, when an immediate recalculation of the LOLP curve is recommended. In the first instance, it is suggested to hold this review annually. Alternatively, a review will be held when there are significant changes to the underlying data (such as new generation). Any change in the LOLP curve should be published prior to the month to which it applies.

5. SEPARATE VARIABLE AND EX-POST LOLP CURVES

The Regulatory Authorities have previously suggested the use of two separate LOLP curves applicable to the Variable and Ex-Post elements of the CPM. The proposal to use two such tables was driven by the timing of the allocation of the Variable and Ex-Post pots. The allocation of the Ex-Post element occurs once all data for the relevant month has been collected – as such the allocation more accurately reflects the value of capacity in each Trading Period than the month-ahead allocation of the Variable element. The Regulatory Authorities considered a “sharper” signal could be merited for the Ex-Post element while a smoothed signal was considered more appropriate for the Variable element.

Section 3.3 and 4.4 have already proposed changes to the LOLP curve that could provide a sharper or smoother signal, i.e. the flattening of the LOLP curve beyond a certain margin and the derivation of separate LOLP curves for planned outages respectively. These options can be used without a fundamental change to the methodology, though there are implications for systems and incentives, particularly in relation to the use of planned outage information. A smoothed signal for the Variable Capacity Payment can be achieved by using a flattened constant LOLP curve, whilst the ‘sharper’ signal for the Ex-Post Capacity Payment could come from *unflattened* variable LOLP curves.

Finally, it should be noted that the approach to determining the ex-ante (and ex-post) margin are equally important as the shape of the LOLP curve in distributing the capacity pot. A smoother signal for the Variable element can also be achieved through more cautious assumptions on the ex-ante margin in terms of load, wind and forced outage[16]. This means that a single Margin-LOLP lookup table can be used for both Variable and Ex-Post Capacity Payments.

[16] For example, ignoring wind generation would reduce the forecast error of the margin.

6. PROPOSALS AND VIEWS INVITED

The Regulatory Authorities are proposing the use of the *stacking* methodology – described in Section 3.2.4 – to derive Loss of Load Probability values, as it is fast, efficient and transparent. This method uses actual data – compiled by the TSOs – on capacity and forced outage probabilities of all units with a controllable capacity. No adjustments will be made to the LOLP curve to account for planned outage, only the (permanent) entry or exit of a unit mid-year would warrant a recalculation. The options of flattening the LOLP curve and using different curves for Variable and Ex-Post Capacity payments were considered by the Regulatory Authorities, but are not being further pursued.

Views on any of the issues raised in this consultation document are requested by 13 March 2007 and should be sent to Paul.Bell@ofregni.gov.uk. The RAs intend to publish all comments received. Those respondents who would like certain sections of their responses to remain confidential should submit the relevant sections in an appendix marked confidential.

APPENDIX A – STACKING METHOD: 3RD AND 4TH GENERATOR

This appendix continues the example in Section 3.1.4 by adding a third and fourth generator. For simplicity, both are assumed to be 100MW with 5% FOP.

Table A.1: Stacking method: Adding a third generator

X	$G_2(x)$	*	FOP_3	+	$G_2(x-c_3)$	*	$(1-FOP_3)$	=	$G_3(x)$
<100	0.002	*	0.05	+	0.000	*	0.95	=	0.0001
100-200	0.040	*	0.05	+	0.002	*	0.95	=	0.0039
200-300	0.088	*	0.05	+	0.040	*	0.95	=	0.0424
300-400	1.000	*	0.05	+	0.088	*	0.95	=	0.1336

Table A.2: Stacking method: Adding a fourth generator

X	$G_3(x)$	*	FOP_4	+	$G_3(x-c_4)$	*	$(1-FOP_4)$	=	$G_4(x)$
<100	0.0001	*	0.05	+	0.0000	*	0.95	=	0.000005
100-200	0.0039	*	0.05	+	0.0001	*	0.95	=	0.000290
200-300	0.0424	*	0.05	+	0.0039	*	0.95	=	0.005825
300-400	0.1336	*	0.05	+	0.0424	*	0.95	=	0.046960
400-500	1.0000	*	0.05	+	0.1336	*	0.95	=	0.176920

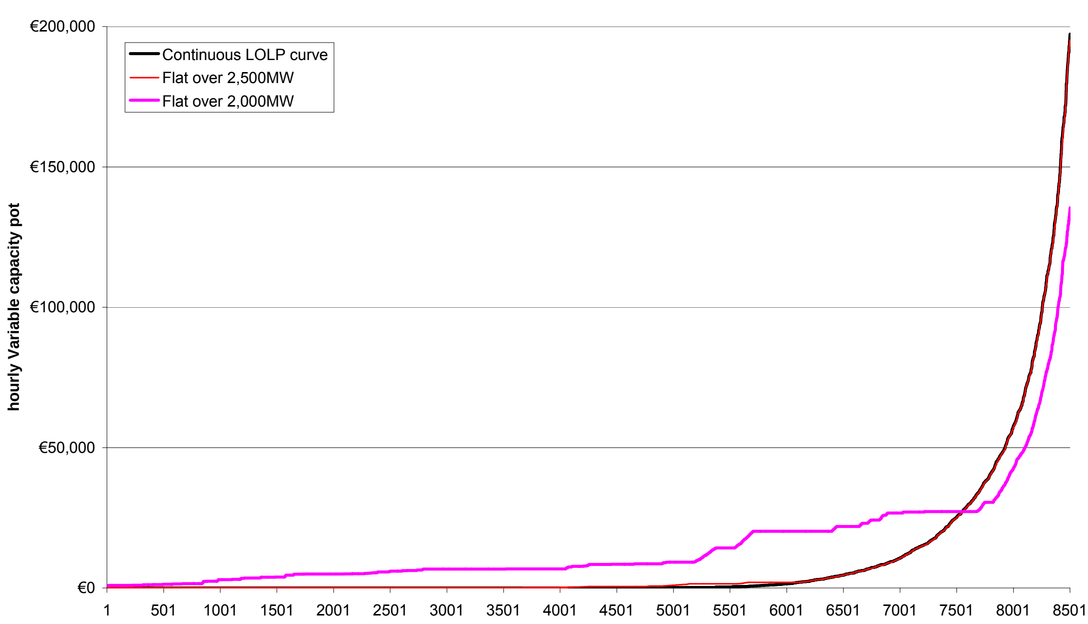
APPENDIX B – FLAT LOLP CURVE AND DISTRIBUTION VCP

The table and graphs below show the distribution of the Variable Capacity Payment Pot over the 8,760 hours in a year (split over two graphs). There is little visual difference between the continuous LOLP curve and that which is flattened for margins over 2,500MW. For illustrative purposes only, an example Pot of €540million is used, of which the Variable component equals 40%.

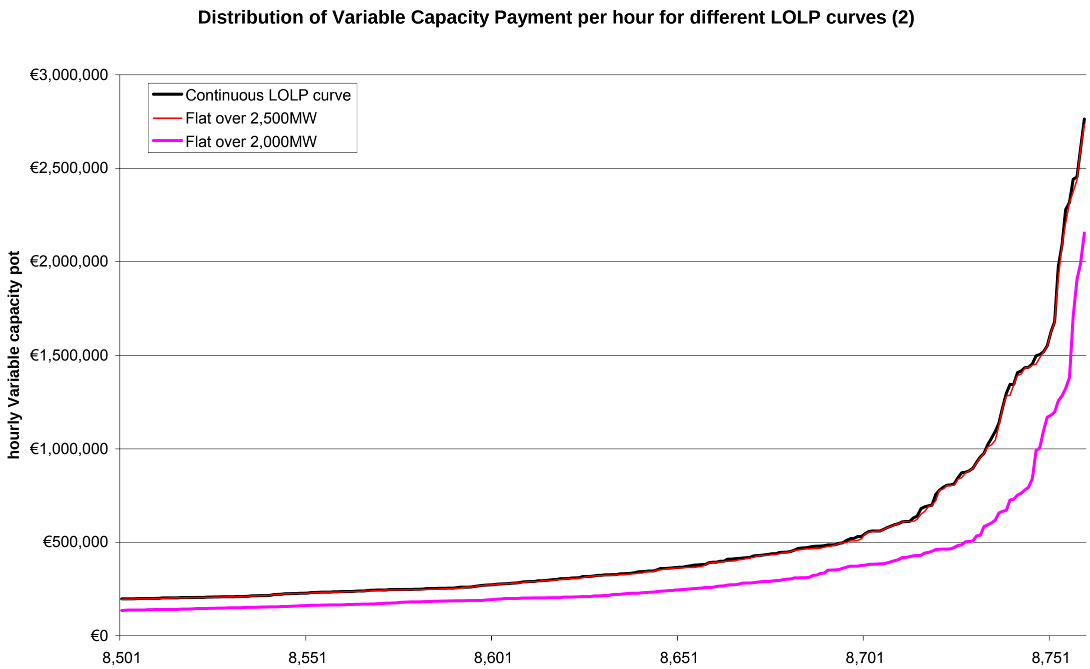
Table B.1: Distribution of hourly Variable Capacity Pot using flattened LOLP curves

Percentiles	*Continuous LOLP Curve*	*Flat after 2,500MW*	*Flat after 2,000MW*
25%	€0	€76	€6,182
50%	€11	€515	€7,335
75%	€6,045	€5,965	€22,515
90%	€51,662	€51,416	€36,666
95%	€118,896	€118,633	€83,125
96%	€142,017	€141,270	€102,756
97%	€179,611	€179,365	€124,028
98%	€248,712	€245,704	€167,145
99%	€385,055	€381,492	€256,726

Distribution of Variable Capacity Payment per hour for different LOLP curves (1)



(continued)


Distribution of Variable Capacity Payment per hour for different LOLP curves (2)
Continuous LOLP curve
Flat over 2,500MW
Flat over 2,000MW
hourly Variable capacity pot
€3,000,000
€2,500,000
€2,000,000
€1,500,000
€1,000,000
€500,000
€0
8,501
8,551
8,601
8,651
8,701
8,751

APPENDIX C – CONSTANT AND VARIABLE LOLP CURVE

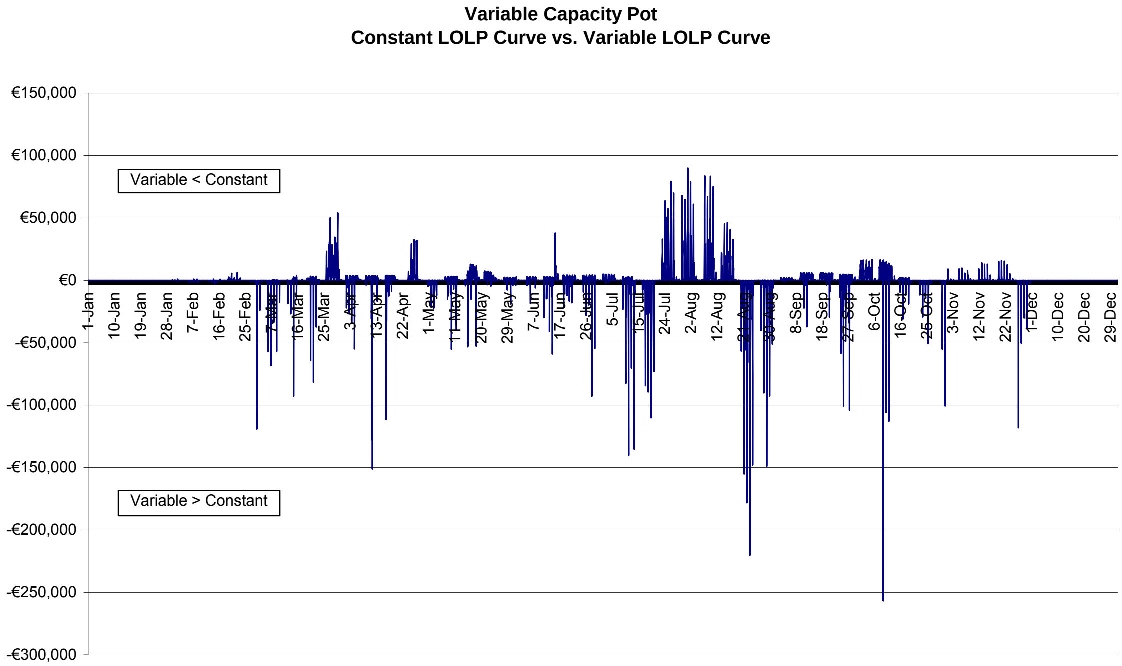


Table C.1: Distribution of hourly Variable Capacity Pot using variable LOLP curves and constant LOLP curve

Percentiles	*Variable LOLP Curves*	*Constant LOLP Curve*
25%	€0	€0
50%	€6	€11
75%	€5,248	€6,045
90%	€47,222	€51,662
95%	€115,571	€118,896
96%	€146,358	€142,017
97%	€194,795	€179,611
98%	€252,313	€248,712
99%	€429,914	€385,055